5. KINEMATICS AND KINETICS

Kinematics :

Kinematics is the branch of engineering mechanic, which deals with the motion of particles and bodies without consideration of forces required to produce them. The term kinetics stands for the displacement, velocity, and acceleratic of a particle (or) of a rigid body.

Rectilinear Motion :

The motion of a body along a striaght line is known as Rectilinear motion (or) Linear motion.

Example : 1. A car moving on a straight road

2. A body projected vertically upwards in the air with a certain velocity

3. A body falling vertically falling downwards.

Curvilinear Motion :

A body is moving along a curved path is known to be in curvilinear motion.

Example : 1. A car moving on a curved road

2. A body is thrown at certain distance etc...

Velocity :

It is defined as the rate of change of displacemer of a body moving in a straight line.

It is measured in m/sec.

Velocity is a vector quantity and is denoted by "$\vec{V}$"

Let

s = distance travelled by the body in a st-line

t = time in seconds

then velocity of the body $V = \frac{s}{t}$ mathematically it can also be written as $\boxed{V = \frac{ds}{dt}}$

Acceleration:

It is defined as the rate of change of velocity of a body. It is denoted by "a" and it is measured in m/sec^2

$$\therefore \boxed{\text{acceleration} = \frac{\text{velocity}}{\text{time}}}$$

It can also be written as $a = \frac{dv}{dt}$ $\left[\text{but } v = \frac{ds}{dt}\right]$

$$a = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}.$$

and also $a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt}$

$$\therefore \boxed{a = v \cdot \frac{dv}{ds}}$$

but here $\frac{ds}{dt} = v$

Equations of motion of a body in a straight line

Let us consider a body which is moving in a straight line.

Let

u = intial velocity in m/sec

v = final velocity in m/sec

a = acceleration in m/sec^2

t = time in seconds during which velocity change from u to v

s = distance travelled by the body in time "t" secon

1. Equation for final velocity:

Change of velocity = final velocity − intial velocity

$= v-u$

the rate of change of velocity $= \frac{v-u}{t}$

but rate of change of velocity is nothing but accelerat

$$a = \frac{v-u}{t}$$

$$\therefore \boxed{v = u+at} \text{ ——— } (1)$$

2. Equation for motion of a body for distance covered:

We know that average velocity $= \frac{v+u}{2}$

$\therefore$ but from eq^n (1) $v = u+at$

$\therefore$ average velocity $= \frac{u+at+u}{2}$

$$\frac{s}{t} = \frac{2u}{2} + \frac{at}{2} \qquad \left[\because \text{velocity } v = \frac{s}{t}\right]$$

$$\frac{s}{t} = u + \frac{1}{2}at$$

$$\therefore \boxed{s = ut + \frac{1}{2}at^2} \text{ --- } ②$$

3.

Now from ①

$$t = \frac{v-u}{a}, \text{ substituting 't in ②}$$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$= \frac{uv-u^2}{a} + \frac{1}{2}\cancel{a}\cdot\frac{1}{a^{\cancel{2}}}(v^2+u^2-2uv)$$

$$= \frac{\cancel{2uv} - 2u^2 + v^2 + u^2 - \cancel{2uv}}{2a}$$

$$s = \frac{v^2-u^2}{2a}$$

$$\therefore \boxed{v^2 - u^2 = 2as}$$

1. A body is moving with a velocity of 2m/sec after 4 sec the velocity of the body becomes 5m/sec. Find the acceleration of the body.

A. Given,

time $t = 4sec$

intial velocity $u = 2\,m/sec$

final velocity $v = 5\,m/sec$

$$a = \frac{v-u}{t} = \frac{5-2}{4}$$

$$\boxed{a = 0.75\, m/sec^2}$$

2. A car is moving with a velocity of 15 m/sec. The car brought to rest by applying brakes in 5sec. determine

i, retaration

ii, distance travelled by the car after brakes.

A. Given,

final velocity = 15 m/sec

time taken $t = 5$ sec

i, $a = \frac{-v-u}{t} = \frac{-15}{5}$ [$\because u=0$]

$$\boxed{a = -3\, m/sec^2}$$

ii, $s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(-3)(5)^2$

$$\boxed{s = 37.5\ m}$$

3. A particle moves along a straight line so that its displacement in metres from a fixed point in given by

$s = t^3 + 3t^2 + 4t + 5$ find

i, velocity at start and after 4 sec

ii, acceleration at start and after 4 sec

A. Given

displacement $s = t^3 + 3t^2 + 4t + 5$

by differentiating the above eqⁿ w.r. to 't'

$$\frac{ds}{dt} = 3t^2 + 6t + 4$$

$$v = 3t^2 + 6t + 4 \qquad \left[\because \frac{ds}{dt} = v\right]$$

by differentiating the above eqn w.r. to "t"

$$\frac{dv}{dt} = 6t + 6$$

$$a = 6t+6 \qquad \left[\because a = \frac{dv}{dt}\right]$$

i, velocity at start $t = 0$

$$v = 3(0)^2 + 6(0) + 4$$

$$\boxed{v = 4\,m/sec}$$

when $t = 4\,sec$

$$v = 3(4)^2 + 6(4) + 4$$

$$\boxed{v = 76\,m/sec}$$

ii, acceleration at $t = 0$

$$a = 6(0) + 6$$

$$\boxed{a = 6\,m/sec^2}$$

at $t = 4\,sec$

$$a = 6(4) + 6$$

$$\boxed{a = 30\,m/sec^2}$$

4. The motion of the particle along a straight line is governed by the relation $a = t^3 - 2t^2 + 7$ where a is the acceleration in m/sec^2 and t is the time in seconds. at time $t = 1\,sec$ the velocity of the particle is $3.58\,m/sec$

and the displacement is 9.39 m. Calculate the displacement velocity and acceleration at time $t = 2$ sec.

A. Given,

$$a = t^3 - 2t^2 + 7 \quad \text{——— (1)}$$

integrating eqⁿ (1) we get

$$\int a = \int t^3 - 2t^2 + 7 \Rightarrow \int \frac{dv}{dt} = \int t^3 - 2t^2 + 7$$

$$\int dv = \int t^3 dt - 2\int t^2 dt + 7\int 1 \cdot dt$$

$$V = \frac{t^4}{4} - 2\frac{t^3}{3} + 7t + c_1 \quad \text{——— (2)}$$

integrating eqⁿ (2) we get

$$\int \frac{ds}{dt} = \int \frac{t^4}{4} - \frac{2t^3}{3} + 7t + c_1$$

$$\int ds = \frac{1}{4}\int t^4 dt - \frac{2}{3}\int t^3 dt + 7\int t\,dt + c_1\int 1\,dt$$

$$s = \frac{t^5}{20} - \frac{2t^4}{12} + \frac{7t^2}{2} + c_1 t + c_2$$

at $t = 1$ sec

$$V = \frac{(1)^4}{4} - \frac{2(1)^3}{3} + 7(1) + c_1$$

$$3.58 = \frac{1^4}{4} - \frac{2}{3} + 7 + c_1$$

$$c_1 = -3$$

$$V = \frac{t^4}{4} - \frac{2t^3}{3} + 7t - 3$$

at $t = 2$ sec

$$V = \frac{(2)^4}{4} - \frac{2(4)^3}{3} + 7(2) - 3$$

$$\boxed{V = 9.66 \ m/sec}$$

at $t = 1sec$

$$S = \frac{t^5}{20} - \frac{2t^4}{12} + 7\frac{t^2}{2} + C_1 t + C_2$$

$$9.39 = \frac{(1)^5}{20} - \frac{2(1)^4}{12} + \frac{7(1)^2}{2} - 3(1) + C_2$$

$$C_2 = 9.01$$

at $t = 2sec$

$$S = \frac{(2)^5}{20} - \frac{2(2)^4}{12} + 7\frac{(2)^2}{2} - 3(2) + 9.01$$

$$\boxed{S = 15.95m}$$

$$a = (2)^3 - 2(2)^2 + 7$$

$$\boxed{a = 7 \ m/sec}$$

5. A bullet moving at the rate of 250 m/sec is fired into a log of wood. The Bullet penerates to a depth of 40cm if the bullet moving with the same velocity is fired into a similar piece of wood of 20cm thickness, with what velocity would it emerge. Takes the resistance to be uniform in both the cases.

A. Case(i):

intial velocity $u = 250 m/sec$

final velocity $v = 0 m/sec$

displacement of bullet $s = 40cm$
$= 0.4m$

$\therefore$ from $v^2 - u^2 = 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{(-250)^2}{2(0.4)}$$

$$\boxed{a = -78125 \ m/sec^2}$$

Case (ii):

intial velocity of bullet $u = 250 \ m/sec$

distance travelled by bullet $s = 20cm = 0.2m$

the resistance is given to be uniform

$\therefore$ acceleration is same $a = -78125 \ m/sec^2$

$$v^2 - u^2 = 2as$$

$$v^2 = 2as + u^2$$

$$v^2 = 2(-78125) \times 0.2 + (250)^2$$

$$v = \sqrt{31250}$$

$$\boxed{v = 176.77 m/sec}$$

6. A tower is 90m height. A particle is dropped from the top of the tower and at the same time another particle is projected upwards from the foot of the tower. Both the particles meet at a height of 30m. find the velocity with which the 2nd particle is projected upwards.

A. Height of the tower $h = 90m$

distance travelled by the particle ① $h_2 = h - h_1$
$= 90 - 30$
$= 60m$

distance travelled by the 2nd particle $h_1 = 30m$

Case(i):

first particle moving vertically downwards.

intial velocity $u = 0 m/sec$

$g = 9.81 m/sec$

distance travelled by particle ① $h_2 = 60m$

$$S = ut + \frac{1}{2}at^2$$

$$h_2 = 0 + \frac{1}{2}gt^2 \quad [\because u = 0]$$

$$\frac{h_2 \times 2}{g} = t^2 = t^2 = \frac{60 \times 2}{9.81}$$

$$t = \sqrt{12.23}$$

$$\boxed{t = 3.49 sec}$$

Case(ii):

partical ② is projected vertically upwards.

intial velocity $u = ?$

$g = 9.81 m/sec$

distance travelled by particle ② $h_1 = 30m$

time $t = 3.49 sec$

$$s = ut + \frac{1}{2}at^2$$

$$h_2 = u(3.49) - \frac{1}{2} \times 9.81 \times (3.49)^2$$

$$30 = 3.49 \times u - 59.74$$

$$u = \frac{89.74}{3.49}$$

$$\boxed{u = 25.71 m/sec}$$

An aeroplane is flying horizontally at a height of 800m a bomb is released from the aeroplane when the speed of the aeroplane is 600 km/hr. determine the time required for the bomb to reach the ground and the horizontal distance travelled by the bomb during flight.

A. Given,

speed of the aeroplane $u = 600\ km/hr$

$$= 600 \times \frac{5}{18} = 166.66\ m/sec$$

height of aeroplane $h = 800m$

Case (i):

Consider the motion of the bomb in the downward direction.

Intial velocity $u = 0$

height $h = 800m$

$g = 9.81\ m/sec^2$

from $s = ut + \frac{1}{2}gt^2$

$$800 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$t^2 = \frac{800 \times 2}{9.81} = 163.09$$

$$\boxed{t = 12.77\,sec}$$

Case (ii):

velocity of bomb = velocity of aeroplane = 166.66 m/sec

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$ (horizontal distance)

horizontal distance = velocity × time

$s = 166.66 \times 12.77$

$\boxed{s = 2128.24\,m}$

3. An aeroplane flying horizontally at a height of 900m. The bomb is released from the aeroplane when the speed of the aeroplane is 700 kmph determine the time required for the bomb to reach the ground and the horizontal distance travelled by the bomb during flight.

A. Given,

height $h = 900m$

speed of aeroplane $= 700$ kmphr

$= 700 \times \frac{5}{18} = 194.44\,m/sec$

Case(i):

bomb is moving in downward direction

intial velocity $u = 0m/sec$

height $h = 900m$

acceleration due to gravity $g = 9.81\,m/sec^2$

from

$$h = ut + \frac{1}{2}at^2$$

$$900 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$t^2 = \frac{900 \times 2}{9.81} = 183.48$$

$$\boxed{t = 13.5\,sec}$$

Case(ii):

velocity of bomb = velocity of aeroplane = 194.44 m/sec

horizontal distance = velocity x time

$s = 194.44 \times 13.5$

$$\boxed{s = 2624.94 m}$$

9. A stone is dropped into a well is heard the water after 4 sec. Find the depth of the well. If the velocity of the sound is 350 m/sec.

A. Given,

time $t = 4 sec$

velocity of sound $= 350 m/sec$

To find the depth of well = ?

t_1 = time taken by the stone to reach bottom of the wel

t_2 = time taken by the sound from the bottom of the well to the top of the well.

Total time $T = t_1 + t_2 = 4 sec$ —— ①

let h = depth of the well

Case (i):

Consider the motion of the stone.

intial velocity $u = 0$

acceleration due to gravity $g = 9.81 m/s$

$t = t_1$

$h = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2} \times 9.81 \times t_1^2$

$t_1^2 = \frac{h}{4.905}$ —— ②

Case(ii):

Consider the motion of sound travelling from bottom of the well to the top of well

$$\text{velocity of sound} = \frac{\text{displacement of sound}}{\text{time}}$$

$$350 = \frac{h}{t_2} \Rightarrow t_2 = \frac{h}{350}$$

from ②

$$t_2 = \frac{4.905}{350} \times t_1^2$$

$$t_2 = 0.140 \times t_1^2$$

from ①

$$4 = t_1 + t_2 \Rightarrow 4 = t_1 + 0.140 \times t_1^2$$

$$0.140 \times t_1^2 + t_1 - 4 = 0$$

$$\boxed{t_1 = 3.798\,sec}$$

$$h = 4.905 \times (3.798)^2$$

$$\boxed{h = 70.753\,m}$$

10. A stone is thrown vertically upwards with a velocity of 19.6 m/sec from the top of a tower 24.5 m height calculate

i, Time required for the stone to reach the ground

ii, Velocity of the stone in its downwards travel at the point in the same level as the point of projection.

iii, The maximum height to which the stone until rise in its flight.

A. Given,

intial velocity of stone $u = 19.6 m/sec$

height of tower $h = 24.5m$

i, Time required for stone to reach the ground

from

$$h = ut + \frac{1}{2} gt^2$$

$$24.5 = 19.6t + \frac{1}{2} \times 9.81 \times t^2$$

Let us 1st calculate the maximum height to which the stone will rise

iii,

Let

$AC = h_1$ = maximum height to which the stone until rise

motion of the stone in the upward direction

$\because$ $u = 19.6 m/sec$

$g = -9.81 m/sec^2$

at the highest point final velocity $v = 0 m/sec$

from eqn $v^2 - u^2 = -2gh_1$

$$h_1 = \frac{v^2 - u^2}{-2g} = \frac{(0)^2 - (19.6)^2}{-2(9.81)}$$

$$\boxed{h_1 = 19.58m}$$

ii, The velocity of the stone when it is moving downwar from "c" at a point "E" which is in the same level as point "A" is equal to the velocity with which the stone is thrown upwards from point "A" in

∴ Velocity at point "E" = 19.6 m/sec

ii, Let t_1 = time for the stone reach the maximum height [i.e to move from A to C]

t_2 = time for the stone to move from C to D

$t = t_1 + t_2$ = Total time for the stone to reach the ground

Stone moving upwards from point A to C:

from $v = u + gt_1$

$0 = 19.6 - 9.81(t_1)$

$\cancel{-}19.6 = \cancel{-}9.81(t_1)$

$\boxed{t_1 = 1.997 \text{ sec}}$

the

rise

Stone moving from point C to D in downward direction:

$g = 9.81$, $u = 0$ m/sec

total distance travelled by stone $CD = CE + ED = 19.58 + 24.5$

$CD = 44.08$ m

$h = ut_2 + \frac{1}{2}gt_2^2$

$44.08 = \frac{1}{2} \times 9.81 \times t_2^2$

$\boxed{t_2 = 2.99 \text{sec}}$

onwar

Total time taken by the stone to reach the ground

as

tone.

$t = 1.997 + 2.99$

$\boxed{t = 4.994 \text{ sec}}$

Kinematics of curvilinear Motion :

Angular velocity:

It is defined as the rate of change of angular displacement. It is always measured in terms of angle covered by the body from the intial position.

Let, a body is moving along a circular path as shown in figure.
Let, initially the body is at "B" and after time "t" the body is at "c".

Let angle $BAC = \theta$

t = time taken by the body

$\therefore$ angular velocity $= \dfrac{\text{angular displacement}}{\text{time}} = \dfrac{\theta}{t}$

in differential form

$$\boxed{\omega = \frac{d\theta}{dt}}$$

2M

Relation between linear velocity and angular velocity

We know that angular velocity $= \dfrac{\theta}{t}$

Let v = linear velocity

$= \dfrac{\text{linear displacement}}{\text{time}}$

but linear displacement = arc BC
$= r \times \theta$

where AB = radius of circle $= r$

$\therefore$ linear velocity $v = \dfrac{\text{linear displacement}}{\text{time}}$

$= \dfrac{r \times \theta}{t} = r \times \dfrac{\theta}{t}$

$v = r \times \omega \quad [\because \theta/t\ \omega]$

where ω = angular velocity

Angular acceleration (α):

It is defined as the rate of change of angular velocity. It is measured in radians/sec^2

Let α = angular acceleration

$$\boxed{\alpha = \frac{\omega}{t}}$$

in differential form

$$\alpha = \frac{d\omega}{dt} \qquad \left[\text{but } \omega = \frac{d\theta}{dt}\right]$$

$$= \frac{d(d\theta)}{dt} = \frac{d^2\theta}{dt^2}$$

(or)

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha = \frac{d\omega}{d\theta} \times \frac{d\theta}{dt} = \frac{d\omega}{d\theta} \cdot \omega$$

$$\boxed{\alpha = \omega \cdot \frac{d\omega}{d\theta}} \qquad \left[\because \frac{d\theta}{dt} = \omega\right]$$

Equations of motion of a body along a curved path:

Let ω_0 = intial angular velocity

ω = final angular velocity

θ = angular displacement

t = time taken by the body

α = angular acceleration

Change of angular velocity = final angular velocity − intial angular velocity

$$= \omega - \omega_0$$

rate of change of angular velocity $= \frac{\omega - \omega_0}{t}$

but rate of change of angular velocity = α = angular accelerati

$$\therefore \alpha = \frac{\omega - \omega_0}{t}$$

$$\alpha t = \omega - \omega_0$$

$$\boxed{\omega = \omega_0 + \alpha t} \quad \text{——— (1)}$$

average angular velocity $= \frac{\text{final angular velocity} + \text{intial angular velocity}}{2}$

$$= \frac{\omega + \omega_0}{2}$$

average angular displacement = angular velocity × time

$$\theta = \omega \times t$$

$$\theta = \left(\frac{\omega + \omega_0}{2}\right) t$$

but from (1)

$$\omega = \omega_0 + \alpha t$$

$$\theta = \left(\frac{\omega_0 + \alpha t + \omega_0}{2}\right)t$$

$$\theta = \frac{2\omega_0 + \alpha t}{2} \times t$$

$$\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2} \quad \text{---} (2)$$

from (1) $\omega = \omega_0 + \alpha t$

$$t = \frac{\omega - \omega_0}{\alpha}$$

substituting t in eq^n (2)

$$\theta = \omega_0\left(\frac{\omega - \omega_0}{\alpha}\right) + \frac{1}{2}\alpha\left(\frac{\omega - \omega_0}{\alpha}\right)^2$$

$$= \frac{\omega\omega_0 - \omega_0^2}{\alpha} + \frac{1}{2}\cancel{\alpha}\cdot\frac{1}{\alpha^{\cancel{2}}}(\omega^2 + \omega_0^2 - 2\omega\omega_0)$$

$$= \frac{\cancel{2\omega\omega_0} - 2\omega_0^2 + \omega^2 + \omega_0^2 - \cancel{2\omega\omega_0}}{2\alpha}$$

$$= \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\boxed{\omega^2 - \omega_0^2 = 2\alpha\theta}$$

Relation between RPM (N) and angular velocity (ω):

Let

N = RPM of the body

ω = angular velocity of the body

No. of revolutions in one minute (or) 60 sec = N

No. of revolutions in one sec = $\frac{N}{60}$

but in 1 revolution the body covers an angel equal to 360° (or) 2π radians

NO. o

angu

$\therefore$ angle covered by the body in 1sec = angle covered in 1 revolution × No. of revolution in 1sec

$$= 2\pi \times \frac{N}{60}$$

$$\boxed{\omega = \frac{2\pi N}{60}}$$

i, An

1. A body is rotating with an angular velocity of 5 radians per second after 4 sec the angular velocity of the body becomes 13 radians/sec determine the angular acceleration of the body.

A. Given,

$\omega_0 = 5$ radians/sec

$\omega = 13$ radians/sec

time $t = 4$ sec

$\therefore$ angular acceleration $\alpha = \frac{\omega - \omega_0}{t} = \frac{13-5}{4}$

ii, ti

$$\therefore \boxed{\alpha = 2 \text{ rad/sec}^2}$$

2. A wheel rotating about a fixed axis 20rpm is uniformly accelerated for 70 sec, during which time it makes 50 revolutions find

i, angular velocity at the end of this interval

ii, time required for the speed to reach 100rpm

A. Given,

$N_0 = 20$ rpm

Intial angular velocity $(\omega_0) = \frac{2\pi N_0}{60} = 2.09$ rad/sec time $t = 70 sec$

No. of revolutions in 70 sec = 50 rpm

angular displacement = angle covered in 1 revolution × No. of revolutions

$\theta = 314$ radians

i, Angular velocity at the end of the given interval

from $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

$\theta = 2.09 \times 70 + \frac{1}{2} \times \alpha \times 70^2$

$314 = 2.097 \times 70 + \frac{1}{2} \times \alpha \times 70^2$

$\boxed{\alpha = 0.068 \text{ rad/sec}^2}$

$\omega = \omega_0 + \alpha t$

$\boxed{\omega = 6.85 \text{ rad/sec}}$

ii, time required for the speed to reach 100 rpm

$\therefore \omega = \frac{2\pi N_0}{60}$

$\omega = 10.46$ rad/sec

from $\omega = \omega_0 + \alpha t$

$t = \frac{\omega - \omega_0}{\alpha}$

$= \frac{10.46 - 6.85}{0.068}$

$t = 2.053$

$= 2min \times 0.53 \times 60$

$\boxed{t = 2 \text{min } 318 \text{ sec}}$

3. A fly wheel is rotating 200 rpm and after 10 sec. it is rotating at 160 rpm. if the retardation is uniform, determine no. of revolution made by the fly wheel and time taken by the fly wheel before it comes to rest from the speed of 200 rpm.

A. Given,

No. of revolutions $N_0 = 200$ rpm

intial angular velocity $\omega_0 = \frac{2\pi N_0}{60} = 20.93$ rad/sec

time $t = 10$ sec

No. of revolution in 10 sec = 160 rpm

$\Rightarrow$ angular displacement θ = angle covered in 1 revolution $\times$ No. of revolution

$$\theta = 1004.8 \text{ rad}$$

$\Rightarrow$ To find the no. of revolutions made by the fly where before it comes to rest from 200 rpm

$\therefore$ intial angular velocity $\omega_0 = 20.93$ rad/sec

final angular velocity $\omega = 0$

$$\boxed{\alpha = -0.419 \text{ rad/sec}^2} \quad \left[\because \alpha = \frac{\omega - \omega_0}{t}\right]$$

using

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

$$\therefore \boxed{\theta = 524.001 \text{ rad}}$$

i) Angular displacement = angle covered in 1 revolution $\times$ No. of revolution

$$\theta = 2\pi N$$

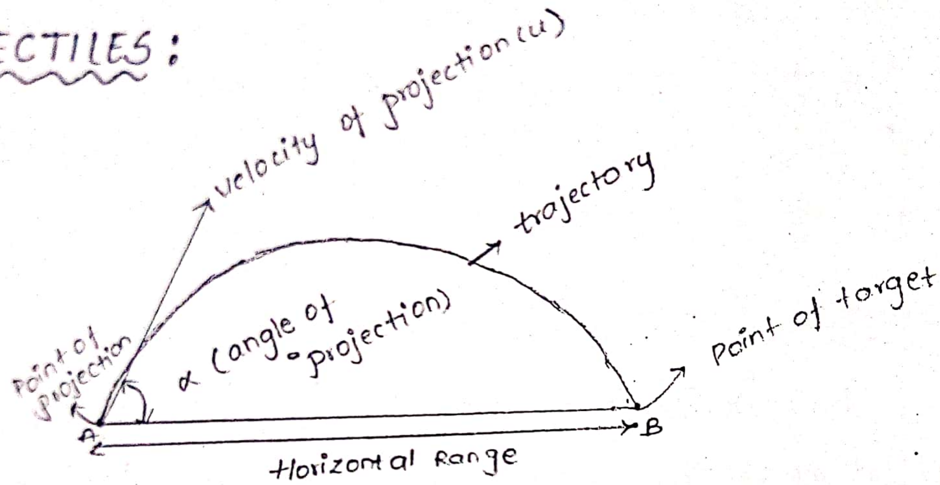
$$\boxed{N = 83.43 \text{ RPM}}$$

∴ time taken by the fly wheel before it comes to rest from 200 rpm

$$\omega = \omega_0 + \alpha t$$

$$\boxed{t = 49.95 \text{ sec}}$$

PROJECTILES:



When a particle is projected upwards at a certain angle (but not vertical i.e. 90°) the particle traces some path in the air and falls on the ground at a point other than the point of projection.

The path traced by the particle in the air is known as trajectory of the particle where as the particle is called "Projectile."

The path traced by the particle is "parabolic."

Velocity of projection:

The velocity with which the projectile is projected into the space is called the velocity of projection. It is denoted by "u".

All definitions

Angle of projection:

It is the angle with the horizontal at which is projected. It is denoted by "α"

Time of flight:

It is the total time taken by a projectile for which the projectile remains in space (or) this is the time interval since the particle is projected and hits the ground. It is denoted by "T."

Horizontal Range:

The horizontal distance between the point of projection and the point where projectile strikes the ground is called horizontal range and is denoted by "R".

Equation for path of a projectile:

Consider a point P on the path traced by a projectile. The velocity of projectile is resloved into two components.

i, horizontal component of velocity $= u\cos\alpha$

ii, Vertical component of velocity $= u\sin\alpha$

Let t = time of projectile from point A to P

x and y are the co-ordinates of 'P'

x = horizontal component of velocity x time $\left[\because v = \frac{x}{t}\right.$

$x = v \times t$

$x = u\cos\alpha \cdot t$ ——— (1)

y=

but

The

also

Max

vertic

zero.

th

$y = u\sin\alpha \cdot t - \frac{1}{2} g t^2$ —— ② $\quad [\because s = ut + \frac{1}{2} g t^2]$

but from ①

$$t = \frac{x}{u\cos\alpha}$$

substituting t in ② we get

$$y = u \cdot \sin\alpha \cdot \frac{x}{u\cos\alpha} - \frac{1}{2} g \left(\frac{x}{u\cos\alpha}\right)^2$$

$$\boxed{y = x\tan\alpha - \frac{1}{2} g \frac{x^2}{u^2\cos^2\alpha}}$$

The equation is known as parabolic equation and it is also known as equation of trajectory.

Maximum height attained by the projectile

The projectile will reach a maximum height when the vertical component of velocity of projection becomes zero.

i.e. $v\sin\alpha = 0$

initial velocity in vertical direction $= u\sin\alpha$

final velocity in vertical direction $= v\sin\alpha = 0$

then

h_{max} = Maximum height attained by the projectile

using the equation $v^2 - u^2 = 2as$.

$$s = h_{max}, \quad a = -g$$

$$(0)^2 - u^2\sin^2\alpha = -2g h_{max}$$

$$\boxed{h_{max} = \frac{u^2\sin^2\alpha}{2g}}$$

Time of flight:

It is the time taken by the projectile in reachin from point A to B as shown in figure.

Let T = time of flight the y coordinate of any point lying on the path of the projectile after a time 't' is given by $y = u\sin\alpha t - \frac{1}{2}gt^2$

at point B,

$$y = 0, \text{ time } t = T$$

$$0 = u\sin\alpha . T - \frac{1}{2}gT^2$$

$$u\sin\alpha . \cancel{T} = \frac{1}{2}gT^{\cancel{2}}$$

$$\boxed{T = \frac{2u\sin\alpha}{g}}$$

Time to reach the highest point:

When the projectile reaches highest point, the fina velocity in vertical direction is zero.

Let T' = time taken by the highest point

we know that $v = u - gt$

$$0 = u\sin\alpha - gT'$$

$$\boxed{T' = \frac{u\sin\alpha}{g}}$$

Horizontal range of projection (R):

It is the horizontal distance between the point of projection and the projectile strikes back to the earth.

i.e. the horizontal AB.

Eqn ① is horizontal range of projectile, denoted by R.

$$x = u\cos\alpha t$$

$\therefore$ R = horizontal component of velocity $\times T$

$$= u\cos\alpha \cdot T$$

$$= u\cos\alpha \cdot \frac{2u\sin\alpha}{g} = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$\boxed{R = \frac{u^2\sin 2\alpha}{g}}$$

from above eqn R will be maximum when $\sin 2\alpha$ is maximum

$$\sin^2\alpha = 1$$

$$\sin 2\alpha = \sin 90^\circ$$

$$\Rightarrow \alpha = 45^\circ$$

$$R_{max} = \frac{u^2\sin 2\alpha}{g}$$

$$R_{max} = \frac{u^2}{g}\sin 90^\circ$$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

1. A particle is projected at angle of 60° with the horizontal. The horizontal range of the projectile is 5 km. Find

i, the velocity of projection

ii, Maximum height attained by the particle.

A. Given,

angle of projection $\alpha = 60°$

Horizontal Range $(R) = 5km$

$= 500m$

i, Velocity of projection

$$\because \quad R = \frac{u^2 \sin 2\alpha}{g}$$

$$u = \sqrt{\frac{Rg}{\sin 2\alpha}} = \sqrt{\frac{500 \times 9.81}{\sin 2(60)°}}$$

$$\boxed{u = 237.98 \ m/sec}$$

ii, Maximum height

$$h_{max} = \frac{u^2 \sin^2\alpha}{2g}$$

$$\boxed{h_{max} = 2164.92 \ m}$$

2. A particle is projected into the air with a velocity of 100 m/sec and at angle of 30° with the horizontal.

find
i, The horizontal range

ii, Maximum height attained by the particle

iii, Time of flight

1. Given,

$u = 100 \, m/sec$

$\alpha = 30^\circ$

i, horizontal range $R = \frac{u^2 \sin 2\alpha}{g}$

$$\boxed{R = 882.79m}$$

ii, Maximum height $h_{max} = \frac{u^2 \sin^2\alpha}{g}$

$$\boxed{h_{max} = 127.42m}$$

iii, Time of flight $T = \frac{2u\sin\alpha}{g}$

$$\boxed{T = 10.19 \text{ sec}}$$

3. A particle is projected at certain angle with horizontal that the horizontal range is 4 times the greatest height attained by the particle. Find the angle of projection.

A. Given,

horizontal range $R = 4h_{max}$

$$R = \frac{u^2 \sin 2\alpha}{g} \quad ; \quad h_{max} = \frac{u^2 \sin^2\alpha}{2g}$$

$$\frac{\cancel{u^2} \sin 2\alpha}{\cancel{g}} = 4 \cdot \frac{\cancel{u^2} \sin^2\alpha}{\cancel{2g}}$$

$$\sin 2\alpha = 2\sin^2\alpha$$

$$\cancel{2} \cancel{\sin\alpha} \cos\alpha = \cancel{2} \sin^{\cancel{2}}\alpha$$

$$\boxed{\alpha = 45^\circ}$$

4. A particle is projected with a velocity of 20 m/sec in air at an angle α with the horizontal. The x & y coordinates of a point lying on the trajectory of the particle with respect to point of projection are 20 m & 8 m respectively. Find the angle of projection of the particle.

A.

A. Given,

Velocity $v = 20$ m/sec

Let α = angle of projection

$\therefore$ from $y = u\sin\alpha \cdot \dfrac{x}{u\cos\alpha} - \dfrac{1}{2} g \left(\dfrac{x}{u\cos\alpha}\right)^2$

$$8 = \cancel{20} \cdot \sin\alpha \cdot \frac{2}{\cancel{20}\times\cos\alpha} - \frac{1}{2}\times 9.81\left(\frac{2}{20\cdot\cos\alpha}\right)^2$$

$$8 = 2\tan\alpha - \frac{0.04905}{\cos^2\alpha}$$

$$8 = 2\tan\alpha - 0.04905\,(1+\tan^2\alpha)$$

$$8 = 2\tan\alpha - 0.04905 - 0.04905\tan^2\alpha$$

$$0.04905\tan^2\alpha - 2\tan\alpha + 8.04905 = 0$$

$$\tan\alpha = 3.273,\ 0.8036$$

$$\boxed{\alpha = 73.0144,\ 38.764^\circ}$$

5. A projectile is fired with a initial velocity of 250 m/s at a target located at a horizontal distance of 4 km & vertical distance of 700m above the gun. determine the value of firing angle to hit the target neglect the air resistance.

A. Given.

intial velocity $u = 250$ m/sec

horizontal distance x (R) = 4km = 4000m

vertical distance y (hmax) = 700m

$$y = u\sin\alpha \cdot \frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$700 = \cancel{250}\sin\alpha \cdot \frac{4000}{\cancel{250}\times\cos\alpha} - \frac{1}{2}\times 9.81\left(\frac{4000}{250\times\cos\alpha}\right)^2$$

$$700 = 4000\tan\alpha - 1255.68\sec^2\alpha$$

$$700 = 4000\tan\alpha - 1255.68(1+\tan^2\alpha)$$

$$1255.68\tan^2\alpha - 4000\tan\alpha + 1955.68 = 0$$

$$= \frac{4000 \pm \sqrt{(4000)^2 - 4(1255.68)(1955.68)}}{2(1255.68)}$$

$$= \frac{4000 \pm 2485.3}{2(1255.08}$$

$$\tan\alpha = 2.582, 0.603$$

$$\alpha = \tan^{-1}(2.582), \tan^{-1}(0.603)$$

$$\boxed{\alpha = 68.82}, \boxed{\alpha = 31.08}$$

6. A ball is thrown top of building with speed 12 m/sec at an angle of dipersion 30° with horizontal, stricks the ground 11.3 m horizontally from foot of the building as shown in fig. determine height of the building.

A. Let,

h = height of buildling

angle of dipersion $\theta = -30°$

velocity of projection $u = 12 m/sec$

Horizontal distance from the building $x = 11.3m$

$$-y = x \tan(-30) - \frac{gx^2}{2u^2}(1 + \tan^2(-30))$$

$$-h = 11.3 \times \tan(-30) - \frac{9.81 \times (11.3)^2}{2 \times (12)^2}(1 + \tan^2(-30))$$

$$= -6.524 - \frac{1252.63}{288}(1 + 0.33)$$

$$= -6.524 - 5.797$$

$$-h = -12.321$$

$$\boxed{h = 12.321m}$$

7. An aeroplane is flying in horizontal direction with a velocity of 540 km/hr and at a height of 2200m as shown in fig. When it is vertically above the point 'A' on the ground, a body is droped from it. The body strike: the ground at point 'B'. Calculate the distance A, B. Also find velocity at 'B' and time taken to reach 'B'.

t

1. velocity of aeroplane $= 540\, km/hr$

$$= 540 \times \frac{5}{18}$$

$$= 150\ m/sec$$

height of aeroplane from $A = -2200m$

Let x = horizontal distance from point A to B

angle of dipersion $\theta = 0°$

from $y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$

$$-2200 = x\tan(0°) - \frac{9.81 \times x^2}{2\times(150)^2}(1+\tan^2(0))$$

$$x = \frac{-2200 \times 2 \times (150)^2}{-9.81}$$

$$\boxed{x = 3176.750m}$$

Considering horizontal motion with constant velocity

$$V = \frac{dist}{time} \Rightarrow time = \frac{dist}{vel}$$

$$\boxed{t = 21.178\ sec}$$

as

Considering vertical motion under gravity

$$V_B^2 - u_B^2 = 2gh$$

here $u_B = 0$ (falling vertical downward)

strike:

B.

$$V_B^2 = 2 \times 9.81 \times 2200$$

$$\boxed{V_B = 207.759\ m/sec}$$

5. A ball is throwon from horizontal level, such that it clears a ball 6m heights, situated at a horizontal distance of 35m as shown in figure. If the angle of projection is 60° with respect to horizontal, what should be the minimum velocity of projection.

A. angle of projection $(\theta) = 60°$

Let u = minimum velocity of projection

height of wall $(h) = 6m = y$

horizontal distance $x = 35m$

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

$$6 = 60.621 - 6008.625 \cdot \frac{1}{u^2}(1+3)$$

$$6 + 6008.625(4) \cdot \frac{1}{u^2} = 60.621$$

$$24034.5 \times \frac{1}{u^2} = 60.621 - 6$$

$$24034.5 \times \frac{1}{u^2} = 54.621$$

$$24034.5 = 54.621\,u^2$$

$$u^2 = 440.023$$

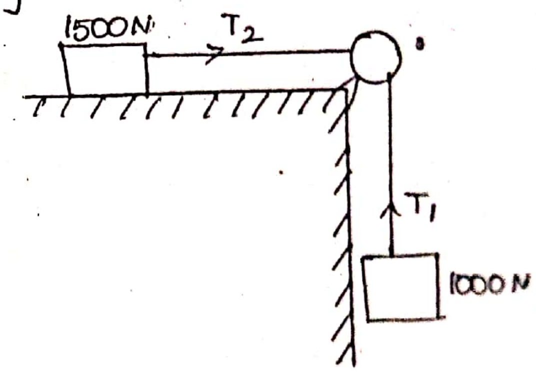
$$\boxed{u = 20.976\,m/s}$$

Kinetics

1. Two bodies of weight 1500N and 1000N are connected to the two ends of a string passing over a rood pulley the weight of 1500N is plane on a roof horizontal surface will the weight 1000N is hanging free in axis. If the co-efficient of kinetic friction is 0.2 at all contact surface

i, Acceleration the system

ii, tension in the string



1. Weight of block 1 = 1000N

Mass of block 1 = 101.936kg

Weight of block 2 = 1500N

Mass of block 2 = 152.905 kg

co-efficient of firction $\mu = 0.2$

a, acceleration of the system

Assume block 1 moving downward direction

We know that

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.20 \times \frac{\pi}{2}}$$

$$T_1 = 1.36910\, T_2 \text{ ——— } (1)$$

Net force acting on block 1

$$W_1 - T_1 = 101.936 \times a \quad (\because F = ma)$$

$$1000 - T_1 = 101.936 \times a \quad \text{—— (2)}$$

$$1000 - 1.36910\, T_2 = 101.936 \times a \quad \text{—— (3)}$$

considering block (2)

Net force $(T_2 - F) = 152.905 \times a$

$$T_2 - 0.20 \times 1500 = 152.905 \times a$$

$$T_2 = 152.905 \times a + 300$$

from (3)

$$1000 - 1.36910 \times 152.905 \times a + 300 = 101.936 \times a$$

$$1000 - 209.3284741 - 410.73 \times a = 101.936 \times a$$

$$379.941 \times a = 110.932 \times a$$

$$a(379.941 - 110.932) = 0$$

$$\boxed{a = 269.009\ m/sec^2}$$

4M ***

D- Alembert's Principle:

Let a body of mass m be moving with a uniform acceleration 'a' under the action of external force 'f' also known as effctive force then according to newtons second law

$$f = ma \text{ ——— (1)}$$

$$f - ma = 0 \text{ ——— (2)}$$

from equation (2) it is clear that by applying the force '-ma' on the body, the body will be in dynamic equilibrium as the sum of all forces acting on the body is zero. such equilibrium is called as Dynamic equilibrium and the force '-ma' is called "D-Alembert's force" (or) "inertia force" (or) "reverse effective force."

"The body will be in equilibrium under the action of external force 'F' and the reversal effective force are inertia force. This is known as D- Alembert's principle."

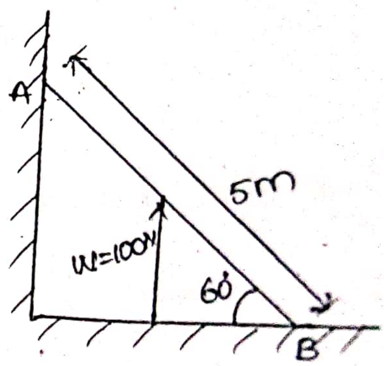
$$\boxed{\text{Inertia force} = \text{mass} \times \text{acceleration}}$$

⇒ Inertia force acts in the opposite direction of the body and passes through the centre of gravity of body.

Ladder friction:

A uniform ladder weight 100N and 5m long has lower end B, resting on the ground and upper end A resting aganist a vertical wall as shown in fig. The inclination of the ladder with horizontal is 60°.

If the co-efficient of friction at all surface of contact is 0.25, determine how much distance up along the ladder a man weight 600N can ascent without causing it to slip?



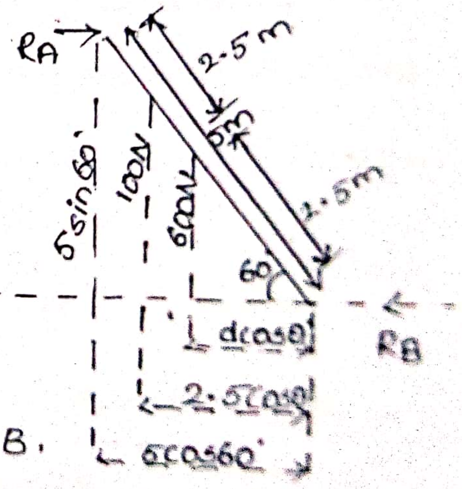
A. Length of the ladder = 5m

weight of the ladder = 100m

man weight = 600N

co-efficient of friction at all contact surfaces $\mu = 0.25$

Angle of inclination of the ladder with the horizontal $\theta = 60°$



Let,

d = distance of man from point B.

Consider the free body diagram the ladder

i, $\Sigma F_x = 0$

$$R_A - F_B = 0$$

$$R_A = 0.25 R_B$$

$$\Rightarrow R_B = 4 R_A$$

ii, $\Sigma F_y = 0$

$R_B + f_A - 100 - 600 = 0$

$4 \times R_A + 0.25 \times R_A = 700$

$\boxed{R_A = 164.705 N}$

$\boxed{R_B = 658.823 N}$

iii, $\Sigma m = 0$

$\Rightarrow 100 \times 2.5 \times \cos 60^\circ + 600 \times d \times \cos 60^\circ - R_A \times 5 \sin 60^\circ - f_A \times 5 \cos 60^\circ$

$0 = 125 + 600 \times 0.5 d - 713.193 - 102.94$

$-600 \times 0.5 d = 125 - 713.193 - 102.94$

$\boxed{d = 2.304 m}$